

# Cherenkov Losses of a Relativistic Neutron in a Conducting Medium\*

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Expressions are derived for the combined Cherenkov and Ohmic losses of a relativistic magnetic dipole moving in a (1) strongly and (2) weakly conducting medium.

## 1. Introduction

The Bohr losses<sup>1</sup> of electrically charged particles in cloud chambers, photographic plates, scintillation counters etc. were summarized by Price<sup>2</sup>, and their Cherenkov losses<sup>3, 4</sup> in a variety of media by Jelly<sup>5</sup>. A unified treatment of the Bohr and Cherenkov losses was first given by Fermi<sup>6</sup>. The combined Cherenkov and Ohmic losses of a charged particle in a conducting medium have been recently studied by us in an earlier paper<sup>7</sup>. The present study is intended to investigate the companion problem of a relativistic neutron and thus extend the earlier works of Frank<sup>8</sup>, Eidman<sup>9</sup>, Ginzburg<sup>10</sup> and Balazs<sup>11</sup> to conducting media. Apart from their intrinsic theoretical interest, studies of radiation in conducting media in motion<sup>12, 13</sup> have acquired renewed significance by virtue of their astrophysical and spatial significance<sup>14–16</sup>.

In view of the duality symmetry<sup>17</sup> inherent in Maxwell's equations in the absence of conductivity, the problem of a neutron can be trivially solved by a mere inspection of the results for a charge<sup>18, 19</sup>. This however is no longer the case in the electrically conducting medium, since it does not possess a magnetic counterpart; and the problem has thus to be examined de novo. An investigation from the rest frame of the medium suffers from the drawback that a magnetic dipole appears to acquire an additional electric dipole moment by virtue of its motion<sup>11</sup>. By the same token, however, the rest frame of the particle also entails its own complications stemming from an involved constitutive matrix<sup>18</sup> of a moving medium, which now acquires

a uniaxial magnetoelectric character<sup>20, 21</sup>. Added to this the conductivity of the medium worsens the situation, since a conducting medium loses its neutrality and appears to be charged when set in motion in an electric field<sup>22</sup>. All these difficulties are however offset by the enormous simplification that the fields become static in the rest frame of the particle, which permits the losses to be derived from a three-fold Fourier integral.

We therefore prefer to work in the rest frame of the neutron and obtain the losses as the work done on it by the fields set up by the polarization of the medium<sup>23</sup>. We first derive the losses of a magnetic monopole and from these deduce the losses of a neutron whose magnetic moment can be looked upon as due to a juxtaposition of two monopole moments<sup>19</sup>. Neat expressions could be obtained for the two realistic limiting cases of a strongly and a weakly conducting medium.

## 2. Fourier Synthesis of the Monopole Fields

Let the point monopole (m) move with a uniform velocity  $\beta c$  along the  $x_1^0$  axis in a homogeneous isotropic neutral conducting medium, with real values of the permittivity ( $\epsilon$ ), permeability ( $\mu$ ) and conductivity ( $\sigma$ ). In the rest frame of the particle, the Maxwell's equations take the form

$$\begin{aligned}\partial_\mu D_\mu(x_\lambda) &= 0, & e_{\mu\nu\varrho} \partial_\nu E_\varrho(x_\lambda) &= 0, \\ \partial_\mu B_\mu(x_\lambda) &= m \delta(x_\lambda), \\ e_{\mu\nu\varrho} \partial_\nu H_\varrho(x_\lambda) &= \frac{1}{c} [J_\mu(x_\lambda) + \beta_\mu \varrho_p(x_\lambda)],\end{aligned}\quad (1)$$

where  $J_\mu$  is the conduction current density and  $\beta_\mu \varrho_p$  is the convection current density due to the apparent charge density  $\varrho_p$  in the relativistically moving conducting medium. The static fields can be resolved into their Fourier components as

$$D_\mu(x_\lambda) = \int D_\mu \exp i k_\lambda x_\lambda d^3k, \text{ etc.} \quad (2)$$

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The constitutive relations of the moving conducting medium can be obtained by a Lorentz transformation of the covariant material tensors  $T_{ijkl}$ <sup>18</sup> and  $\sigma_{ijk}$  that connect the Fourier components of the induction tensor  $H_{ij}$  and the current density four vector  $J_i$  respectively with the field tensor  $F_{ij}$ . These are then given by

$$\begin{aligned} D_1 &= \varepsilon_1 E_1, & D_2 &= \varepsilon_2 E_2 + \xi B_3, & D_3 &= \varepsilon_2 E_3 - \xi B_2, \\ H_1 &= \lambda_1 B_1, & H_2 &= \lambda_2 B_2 + \xi E_3, & H_3 &= \lambda_2 B_3 - \xi E_2, \\ J_1 &= \eta E_1, & J_2 &= \eta E_2 + \Psi B_3, & J_3 &= \eta E_3 - \Psi B_2, \end{aligned} \quad (3)$$

and

$$\varrho_p = -\Psi E_1,$$

where

$$\begin{aligned} \varepsilon_1 &= \varepsilon, & \varepsilon_2 &= \frac{r^2}{\mu} (n^2 - \beta^2), & \lambda_1 &= \frac{1}{\mu}, \\ \lambda_2 &= \frac{r^2}{\mu} (1 - \beta^2 n^2), & \xi &= \frac{\beta r^2}{\mu} (n^2 - 1), \\ \eta &= \sigma r, & \Psi &= \sigma \beta r, & n^2 &= \varepsilon \mu. \end{aligned}$$

Substituting (2) and (3) in (1), we obtain

$$H_1(x_\lambda) = \frac{i m}{8 \pi^3} \int \frac{a^2}{\mu} \cdot \frac{k_1 \exp i k_\lambda x_\lambda}{k_2^2 + k_3^2 - a^2 k_1^2 - 2 i \chi k_1} d^3 k \quad (4a)$$

$$- \frac{m}{4 \pi^3} \int \frac{\chi}{\mu} \cdot \frac{\exp i k_\lambda x_\lambda}{k_2^2 + k_3^2 - a^2 k_1^2 - 2 i \chi k_1} d^3 k \quad (4b)$$

where  $a^2 = r^2(\beta^2 n^2 - 1)$  and  $\chi = (\sigma \beta r \mu)/c$ . In the limit of a non-conducting medium,  $\chi \rightarrow 0$ , (4b) vanishes and (4a) becomes the dual of the  $E_1$  for an electric charge.

### 3. Energy Losses of a Magnetic Monopole

Since  $H_1$  is unaltered by the special Lorentz transformation, we obtain the energy loss of a magnetic monopole as

$$W = - \frac{i m^2}{8 \pi^3} \int_{-\infty}^{\infty} \frac{a^2}{\mu} k_1 dk_1 \int_{-\infty}^{\infty} dk_2 \quad (5a)$$

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{dk_3}{k_3^2 + k_2^2 - a^2 k_1^2 - 2 i \chi k_1} \\ & + \frac{m^2}{4 \pi^3} \int_{-\infty}^{\infty} \frac{\chi}{\mu} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} \frac{dk_3}{k_3^2 + k_2^2 - a^2 k_1^2 - 2 i \chi k_1}. \end{aligned} \quad (5b)$$

Since (5a) and (5b) have the same denominator, the poles of their integrands in the complex  $k_3$  plane are identical for both the integrals. The position of these poles depends on the sign of  $k_1$ <sup>24</sup> and the magnitude of  $k_2$ , in addition to the magnitude of  $\sigma$ . For  $k_1 > 0$ , the two poles lie in the first and third quadrants and are symmetric about the origin, while for  $k_1 < 0$ , they lie in the second and the fourth quadrants. For  $k_2^2 < a^2 k_1^2$  they lie close to the real axis for small values of  $\sigma$ . Due to the presence of  $k_1$  in the numerator of (5a) the contributions of the two poles get added up and lead to a finite energy loss that can be identified as Cherenkov emission. Since  $k_1$  does not occur in the numerator of (5b) the contributions of the two poles cancel one another exactly as  $\sigma \rightarrow 0$ . For  $k_2^2 > a^2 k_1^2$  the two poles lie close to the imaginary axis and their contributions cancel one another in (5a), but get added up in (5b), leading to a divergent expression for the  $k_2$  integral. However, the presence of  $\chi$  in the numerator of (5b) saves the situation, since it can be shown from an examination of the integral that its product with  $\chi$  vanishes as  $\chi \rightarrow 0$ . On the other hand, in a strongly conducting medium, the poles lie always midway between the real and the imaginary axes, and the presence of  $\chi$  in the numerator renders (5b) much the predominant term.

We thus obtain the energy loss of a magnetic monopole in a strongly conducting medium as

$$W^h = \frac{m^2}{4 \pi^2 c^2} \int \varepsilon \left(1 - \frac{1}{\beta^2 n^2}\right)^{1/2} \left[ \frac{2}{\tau} \left(1 + \frac{\omega_h^2}{\omega^2}\right) \right]^{1/2} \cdot \omega^{1/2} d\omega, \quad (6)$$

and in a weakly conducting medium as

$$W^l = \frac{m^2}{4 \pi^2 c^2} \left\{ \int \varepsilon \left(1 - \frac{1}{\beta^2 n^2}\right) \omega d\omega + \int \frac{\varepsilon}{\pi \tau} \cdot \left\{ \left(1 + \frac{\omega^2}{\omega_l^2}\right)^{1/2} - 1 \right\} d\omega \right\}. \quad (7)$$

In the above expressions,  $\tau$  is the relaxation time of the conducting medium and  $\omega_h$  and  $\omega_l$  are the two limiting frequencies characteristic of the medium below and above which the medium can be classed as strongly or weakly conducting respectively.

For an insulating medium  $\tau \rightarrow \infty$ , and the second term in (7) vanishes, leaving the Cherenkov losses given by the first term. It is thus obvious that the two terms in (7) refer respectively to the Cherenkov and Ohmic losses. Such a division is absent from (6) as is to be expected, for, in a strongly con-

ducting medium, any Cherenkov emission that takes place is rapidly absorbed and the entire loss partakes an Ohmic character.

#### 4. Energy Losses of a Magnetic Dipole

An additional parameter, viz. the inclination of the dipole to its line of motion, makes its appearance in the dipole Cherenkov effect. However, the total loss can be obtained by summing up the two losses in the parallel and perpendicular cases<sup>19</sup>. The general expression for the dipole Cherenkov losses is given by<sup>19</sup>

$$W = M_\mu M_\nu \partial_\mu \partial_\nu \left( \frac{H_1}{m} \right), \quad (8)$$

where  $\mathbf{M}$  is the dipole moment vector and  $H_1$  is the magnetic field due to the magnetic monopole. Using Eq. (4) and proceeding analogously, we obtain

$$W_{\parallel}^h = \frac{M_{\parallel}^2}{4 \pi^2 \beta^2 r^2 c^4} \int \varepsilon \left( 1 - \frac{1}{\beta^2 n^2} \right)^{1/2} \left[ \frac{2}{\tau} \left( 1 + \frac{\omega_h^2}{\omega^2} \right) \right]^{1/2} \cdot \omega^{5/2} d\omega, \quad (9a)$$

$$W_{\parallel}^l = \frac{M_{\parallel}^2}{4 \pi \beta^2 r^2 c^4} \left[ \int \varepsilon \left( 1 - \frac{1}{\beta^2 n^2} \right) \omega^3 d\omega + \int \frac{\varepsilon}{\pi \tau} \left\{ \left( 1 + \frac{\omega^2}{\omega_1^2} \right)^{1/2} - 1 \right\} \omega^2 d\omega \right], \quad (9b)$$

for the energy loss of a magnetic dipole moving parallel to its axis, and

$$W_{\perp}^h = \frac{M_{\perp}^2}{12 \pi^2 c^4} \int \varepsilon^2 \mu \left( 1 - \frac{1}{\beta^2 n^2} \right)^{3/2} \frac{1}{\sqrt{2} \tau} \left( 1 + \frac{\omega_h^2}{\omega^2} \right)^{3/2} \cdot \omega^{5/2} d\omega, \quad (10a)$$

$$W_{\perp}^l = \frac{M_{\perp}^2}{8 \pi c^4} \int \varepsilon^2 \mu \left( 1 - \frac{1}{\beta^2 n^2} \right)^2 \omega^3 d\omega \quad (10b)$$

for the energy loss of a magnetic dipole moving perpendicular to its axis in a strongly and a weakly conducting medium respectively.

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